

XXII. *On the Georgian Planet and its Satellites.**By William Herschel, LL.D. F. R. S.*

Read May 22, 1788.

IN a Paper, containing an account of the discovery of two fatellites revolving round the Georgian planet, I have given the periodical times of these fatellites in a general way, and added that their orbits made a considerable angle with the ecliptic. It is hardly necessary to mention, that it requires a much longer series of observations, to settle the mean motions of secondary planets with accuracy, than I can hitherto have had an opportunity of making; but since it will be some satisfaction to astronomers to be acquainted with several of the most interesting particulars, as far as they can as yet be ascertained, I shall communicate the result of my past observations; and believe that, considering the difficulty of measuring objects which require the utmost attention even to be at all perceived, the elements here delivered will be found to be full as accurate as we can at this time expect to have them settled.

The most convenient way of determining the revolution of a fatellite round its primary planet, which is that of observing its eclipses, cannot now be used with the Georgian fatellites, as will be shewn when I come to give the position of their orbits; and as to taking their situations in many successive oppositions of the planet, which is likewise another very eligible method, that must of course remain to be done at proper

per opportunities. The only way then left, was to take the situations of these satellites, in any place where I could ascertain them with some degree of precision, and to reduce them afterwards by computation to such other situations as were required for my purpose.

In January, February, and March, 1787, the positions were determined by causing the planet to pass along a wire, and estimating the angle a satellite made with this wire, by a high magnifying power; but then I could only use such of these situations where the satellite happened to be either directly in the parallel of declination, or in the meridian of the planet; or where, at least, it did not deviate above a few degrees from either of them; as it would not have been safe to trust to more distant estimations. In October I had improved my apparatus so far as to measure the positions by the same angular micrometer with which I have formerly determined the relative positions of double stars*.

In computing the periods of the satellites I have contented myself with synodical appearances, as the position of their orbits, at the time when the situations were taken from which these periods are deduced, was not sufficiently known to attempt a very accurate sidereal calculation. By six combinations of positions at a distance of 7, 8, and 9 months of time, it appears that the first satellite performs a synodical revolution round its primary planet in 8 days 17 hours 1 minute and 19,3 seconds. The period of the second satellite deduced likewise from four such combinations, at the same distance of time, is 13 days 11 hours 5 minutes and 1,5 seconds. The

* For a description of this instrument, see *Phil. Trans.* Vol. LXXI. p. 500. and Vol. LXXV. p. 46.

combinations of which the above quantities are a mean do not differ much among themselves; it may therefore be expected that these periods will come very near the truth; and, indeed, I have for many months past been used to calculate the places of the satellites by them, and have hitherto always found them in the situations where these computations gave me reason to expect to see them.

The epochæ, from which astronomers may calculate the positions of these satellites, are October 19, 1787; for the first 19 h. 11' 28"; and for the second 17 h. 22' 40". They were at those times $76^{\circ} 43'$ north-following the planet; which, as will be shewn in the sequel, is the place of the greatest elongation of the second satellite; where, consequently, its real angular situation is the same as the apparent one. And I have brought the first satellite to the same place, as hitherto there has not been time to discriminate the situation of its orbit from that of the second.

The next thing to be determined in the elements of these satellites is their distance from the planet; and as we know that, when the periodical times are given, it is sufficient to have the distance of one satellite in order to find that of any other, I confined my attention to the discovery of the distance of the second. As soon as I attempted measures, it appeared, that the orbit of this satellite was seemingly elliptical; it became therefore necessary, in order to ascertain its greatest elongation, to repeat these measures in all convenient situations; the result of which was, that on the 18th of March, at 8 h. 2' 50", I found the satellite at the distance of $46''.46$; this being the largest of all the measures I have had an opportunity of taking. Hence, by computation, it appears, that the

satellite's greatest visible elongation from its planet, at the mean distance of the Georgium sidus from the earth, will be $44'',23$.

It ought to be mentioned, that in the reduction of this measure I have used MAYER's tables for the sun, and the tables published in the *Connoissance des Temps* of the year 1787, reduced to the time of Greenwich, for the Georgian planet.

Very possibly this distance might not be taken exactly at the time when the satellite happened to be at the vertex of the transverse axis of its apparently elliptical orbit; but, from other measurements, we have reason to conclude, that it could not be far from that point. For instance, the 9th of November, at 15 h. $56' 15''$, by a mean of four good measures, the satellite was $44'',89$ from the planet; which, by calculation, reduced to the same distance of the Georgium sidus from the earth as the former, gives $41'',33$. And likewise, the 19th of March, at 7 h. $45' 59''$, the distance measured $44'',24$; which, computed as before, gives $42'',15$. Now, we find, when the places are calculated in which the satellite happened to be at the times when these two measures were taken, that they fall on different sides of the former measure, and also on opposite parts of the satellite's orbit; but that nevertheless they agree sufficiently well with the position of the transverse axis which we have adopted in the sequel.

Admitting, therefore, at present, that the satellite moves in a circular orbit about its planet, we cannot be much out in taking the calculated quantity of $44'',23$ for the true measure of its distance. And, having ascertained this point, we calculate, by the law of KEPLER, and the assigned period of the first satellite, that its distance from the planet must be $33'',09$. I ought however to remark, that, in this computation, a true sidereal period should have been used; but, as that cannot as yet

yet be had, the trifling inaccuracy thence arising may well be excused, till, at some future opportunity, we may be permitted to repeat these calculations in a more rigorous manner.

As we are now upon the subject of such parts of the theory of planets as may be determined by calculation, it will not be amiss to see how the quantity of matter and density of our new planet will stand, when compared with the tables that have been given of the same in the other planets; and in order to this, let us admit the following *data* as a foundation for our computation.

The parallax of the sun $8'',63$.

The parallax of the moon $57' 11''$.

Its sidereal revolution round the earth $27 \text{ d. } 7 \text{ h. } 43' 11'',6$.

The mean distance of the Georgian planet from the sun 19,0818.

The mean distance of its second satellite from the planet $44'',23$.

The periodical time of this satellite $13 \text{ h. } 11 \text{ d. } 5' 1'',5$.

Hence we find, that a spectator, removed to the mean distance of the Georgian planet from the earth, would see the radius of the moon's orbit under an angle of $27'',1866$; and if $1, d, t$, represent the quantity of matter in the earth, the distance of the moon, and its periodical time; M, D, T , be made to stand for the same things in our new planet and its second satellite, we obtain, by known principles, $M = \frac{t^2 D^3}{T^2 d^3}$. And, consequently, the quantity of matter in the Georgian planet is to that contained in the earth as 17,740612 to 1.

In order to calculate the density, I compare the mean of the four bright measures of the planet's diameter $3'',7975$ to the

mean of the two dark ones $4'',295$; as they are given in my Paper on the diameter and magnitude of the Georgium fidus, printed in Volume LXXIII. of the Philosophical Transactions, p. 9. 11, 12, 13. Whence we obtain another mean diameter $4'',04625$; which is probably the most accurate of any that we have hitherto ascertained. And let us suppose this measure to belong to the situations of the earth and of the new planet as they were at 10 o'clock, the 25th of October, 1782; which is about the middle of the several times when those measures from which this is deduced were taken. Then, by the tables already referred to, we compute the distance of the two planets from the sun and the angle of commutation; whence, by trigonometry, we find the distance of our new planet from the earth for the supposed 25th of October; and thence deduce its mean diameter, which is $3'',90554$. This, when brought to what it would appear if it were seen from the sun at the earth's mean distance, gives $1' 14'',5246$; which, compared with $17'',26$, the earth's mean diameter, is as 4,31769 to 1. The Georgium fidus, therefore, in bulk, is 80,49256 times as large as the earth; and consequently its density less than that of the latter in the ratio of ,220401 to 1.

To these particulars, though many of them may be of no other use than merely to satisfy our curiosity, we may also add, that the force of gravity, on this planet's surface, is such as will cause an heavy body to fall through 18,67308 feet in one second of time.

It remains now only, in order to complete our general idea of the Georgian planet, to investigate the situation of the orbits of its satellites. I have before remarked, that when I came to examine the distance of the second, I perceived immediately that its orbit appeared considerably elliptical. This induced

me to attempt as many measures as possible, that I might be enabled to come at the proportion of the axes of the apparent ellipsis; and thence argue its situation. But here I met with difficulties that were indeed almost insurmountable. The uncommon faintness of the satellites; the smallness of the angles to be measured with micrometers which required light enough to see the wires; the unwieldy size of the instrument, which, though very manageable, still demanded assistant hands for its movements, and consequently took away a great share of my own directing power, a thing so necessary in delicate observations; the high magnifiers I was obliged to use by way of rendering the spaces and angles to be measured more conspicuous; in short, every circumstance seemed to conspire to make the case a desperate one. Add to this, that no measure could possibly succeed which had not the most beautiful sky in its favour; and we may easily judge how scarce the opportunities of taking such measures must be in the variable climate of this island. As far then as a small number of select measures will permit, which, out of about twenty-one that were taken, amounts only to five, I shall enter into our present subject of the position of the second satellite's orbit.

The following table contains in the first column the correct mean time when the measures were taken. The second gives the quantity of these measures. In the third column are the same measures reduced to the mean distance of the Georgian planet from the earth. The fourth contains the calculated positions of the satellite as it would have appeared to be situated if it had moved in a circular orbit at rectangles to the visual ray; and the degrees are numbered from the first observation supposed to have been at zero, and are carried round the circle from right to left.

March

	D.	H.	'	"	"	"	°	'
March 18	8	2	50	46,46	44,23	0	0	
19	7	47	59	44,24	42,15	26	28	
20	7	44	8	40,23	38,37	53	8	
April 11	9	18	27	35,32	34,35	28	3	13
Nov. 9	15	56	15	44,89	42,88	199	59	

In the use of this table I shall partly content myself with the construction of a figure, and only apply calculation to the most material circumstances. By the third column we see that $44'',23$ is the greatest, and $34'',35$ the least, distance of the satellite. Let therefore an ellipsis be drawn Tab. V. fig. 1. having the transverse and conjugate diameters cp and cv , in the proportion of the above-mentioned measures. About the center c , with the radius cp , describe the circle PSFN; and set off the points March 18, 19, 20. April 11. and Nov. 9. according to the tabular order of degrees, beginning at p , the supposed zero. From these points to the transverse draw the ordinates March 19 s , 20 t , April 11 x , Nov. 9 y . Then, if the satellite moved in a circular orbit at rectangles to the visual ray, we should have seen it at the time given in the table, as the points are placed in the circumference of our circle; but, supposing the plane of the orbit inclined to the visual ray, these points will be projected in the direction of the ordinates; and, falling on the places $pnmro$, will form the ellipsis we have delineated. Now, on comparing the tabular measures of the third column with the distances of $pnmr$ and o from the center c , we find, that they agree full as well as we could expect; and thus, as far as a few observations can do, these measures establish the truth of the above hypothesis.

That we may have a point in our ellipsis from which to depart, I shall have recourse to two measures of positions. The first was taken October 14 d. 16 h. 28' 42'', when the satellite was $66^{\circ} 3'$ south-following the planet. In fig. 2. let qmv be a portion of an ellipsis, constructed on the semi-transverse cq , and semi-conjugate cv , taken as 44,23 to 34,35; qMF an arch of a circle, described with the radius cq about the center c ; m the situation of the satellite in its elliptical orbit, the 14th of October; A its apparent, and M its real place in the circle; Fc the parallel of the planet. Then we shall have, by calculating from the known period, the arch qM $45^{\circ} 17'$; and FA , by observation, $66^{\circ} 3'$. But from the nature of the ellipsis, as Vc is to vc so is Mn (the tangent of the angle qcM to the radius cn) to mn (the tangent of the angle qcA to the same radius). Hence qcA is found $38^{\circ} 7'$; and therefore AcM $7^{\circ} 10'$. That is, when the angle of position was taken, the satellite appeared to be $7^{\circ} 10'$ less advanced in its orbit than it should have done, owing to its motion in an orbit whose plane is inclined to the visual ray. The measure therefore corrected, or rather reduced to the circle, instead of $66^{\circ} 3'$, will be $58^{\circ} 53'$ south-following; to which, adding the calculated arch qA , and from the sum deducting 90° , we have the position qcS with the meridian $14^{\circ} 10'$ on the south-preceding side. In the same manner I proceed with the second measure taken October 20 d. 16 h. 7' 34''; when the satellite appeared to be $82^{\circ} 12'$ north-preceding the planet. Here the arch qM is $25^{\circ} 21'$, AcM $5^{\circ} 9'$; and the measure corrected $77^{\circ} 3'$ north-preceding, which gives the inclination of the axis to the meridian $12^{\circ} 24'$ on the north-following side. I have no reason to prefer either of the measures, and therefore take a mean of both, which is $13^{\circ} 17'$ from south-preceding to

north-following the meridian, as probably nearest the truth; and this position of the axis we may suppose to belong to a time which is about the mean of those from which it has been deduced; or October, 17 d. 16 h.

We are, in the next place, to find the angle which the plane of the meridian made at that time with the plane of the orbit of the Georgian planet. To this end we calculate its longitude and latitude for the given time. Then, in fig. 3. where $\in$ NE is part of the solstitial colure, nc a portion of the orbit, and $\in s$ of the ecliptic; there is given the arch NE, $23^{\circ} 28'$; Ec, $89^{\circ} 27' 49''$,² the complement of the planet's latitude; and the angle $\in Es$, $27^{\circ} 0' 52''$,² or planet's distance from Cancer. By these we find the angle EcN between the circle of latitude Nc and the meridian Ec $11^{\circ} 11' 41''$. Now, let c , in fig. 4. be the place of the Georgian planet, and G $\in$ c a part of its orbit; $e\in s$ part of the ecliptic; Nc the meridian; $\in$ the place of the planet's ascending node; pcq the position of the axis of the apparently elliptical orbit of the second satellite; EcN the angle of position of the Georgium sidus. Then, by calculation, for the above-mentioned day we have $\in c$, the planet's distance from the node on its orbit $44^{\circ} 8' 17''$; $c\in s$ the inclination of its orbit to the ecliptic $46' 13''$; and $\in sc$ a right-angle; whence Ec $\in$ $90^{\circ} 33' 10''$,¹ the supplement of the angle $sc\in$ is found; from which, taking the angle of position NcE, before obtained, we have the remaining angle, NcG, $79^{\circ} 21' 29''$,¹; or inclination of the planet's orbit to the plane of the meridian, which was required.

From the proportion of the transverse cp , fig. 1. to the conjugate cv , we calculate the angle vpc , which may be either acute or obtuse. For here I must take notice, that observations cannot immediately determine whether the satellite, in passing from p through nmv to q , be in the farthest or nearest part of

its orbit; as we shall presently shew that this orbit is not in a situation to permit the fatellite to suffer either eclipses or occultations for some time to come. The angle vpc , therefore, if the arch pvg be turned towards us will be $129^{\circ} 2' 46'',5$; but, if directed the contrary way, $50^{\circ} 57' 13'',5$. There is one circumstance which will bring on a discovery of this particular, without waiting for eclipses; for if the apparent ellipsis of the fatellite's orbit should contract in a year or two, we may conclude this arch to lie towards the sun; if, on the contrary, it opens, we shall know that the fatellite has passed through one of its nodes about eight or nine years ago; and that, therefore, we must not expect to see it eclipsed for more than thirty years to come.

Now, having already determined the position of the axis pc with respect to the meridian, by adding the angles Ncp and NcG , fig. 4. we obtain pcn , $92^{\circ} 38' 29'',1$; and having also now calculated the ambiguous angle npc , we may resolve the quadrantal triangle pcn , in which the angle cnp gives the inclination of the orbit of the fatellite to the orbit of its planet, which will be $99^{\circ} 39' 48'',9$, if the fatellite be approaching to its ascending node; but $80^{\circ} 20' 11'',1$, if it be lately past the descending one.

In the same triangle we find the side nc , which is either $50^{\circ} 59' 0'',8$ or $129^{\circ} 0' 59'',2$; and taking these quantities, increased by six signs, from the longitude of the planet in its orbit, gives the place of the fatellite's ascending node upon the orbit of the planet, either 8 s. $6^{\circ} 2' 0'',3$, if the preceding arch of the orbit pnn , be concave towards the sun; or 5 s. $18^{\circ} 0' 1'',9$, if it be convex.

These elements obtained, we reduce them to the ecliptic by resolving the triangle nm , in which we have mnn , $46' 13''$;

$n\infty$ the distance of the ascending node of the satellite from the descending node of the planet $6^{\circ} 50' 43'',8$ or $84^{\circ} 52' 42'',1$; and ∞nm , the inclination of the satellite's orbit to that of its planet $99^{\circ} 39' 49'',9$ answering to the former, or $80^{\circ} 20' 11'',1$ to the latter. In consequence of this resolution, we have the place of the ascending node of the satellite upon the ecliptic, $\left\{ \begin{smallmatrix} 5\text{ s. } 18^{\circ} 0' 3'',9 \\ 8 \quad 6 \quad 2 \quad ,3 \end{smallmatrix} \right\}$, and its inclination to the same $\left\{ \begin{smallmatrix} 99^{\circ} 43' 53'',3 \\ 81 \quad 6 \quad 4 \quad ,4 \end{smallmatrix} \right\}$. The orbit being situated so, that when the planet will be in the ascending node of this satellite, which will happen about the year $\left\{ \begin{smallmatrix} 1799 \\ 1818 \end{smallmatrix} \right\}$, the northern half of it will be turned towards the $\left\{ \begin{smallmatrix} \text{East} \\ \text{West} \end{smallmatrix} \right\}$ at the time of its meridian passage.

In justice to the foregoing calculations I should add, that the result of them must be considerably affected by any small alteration in the measures upon which they are founded; the general theory, however, will certainly stand good, and a greater perfection in particulars could not have been obtained, unless I had waited some years, at least, in order to multiply good observations. But with objects that are out of the reach of common telescopes, and which therefore cannot be much attended to, even by our most assiduous astronomers, a general theory will perhaps nearly answer all the ends that may be required of it.

The measures of the distances were taken by a good parallel-wire micrometer, contrived so that one of the wires, which is moveable, can pass over the other; by which means central measures may be obtained with more accuracy than by allowing
for

for the thickness of the wires, the ascertaining of which is liable to some difficulties in other constructions; but here, as we can note the divisions on the first appearance of light at either side of the fixed wire, when the moveable one passes over it backwards and forwards, we may very conveniently determine that part of the scale to which the zero ought to answer in central measures. The value of the scale was ascertained by the transit of stars over the two wires opened to a certain number of divisions, and a chronometer beating five times in two seconds of mean time; and in a number of several sets of experiments, the mean of each seldom differed so much as the goodth part of a second of space for each division, and these are large enough to be sub-divided and read off, with good exactness to tenths; and yet the space answering to each part amounts only to 282 millesimals of a second. The measures of the distances also were as often repeated as the opportunities would permit, and a mean of them has been used.

The light of the satellites of the Georgian planet is, as we may well expect, on account of their great distance, uncommonly faint. The second is the brightest of the two, but the difference is not considerable; besides, we must allow for the effect of the light of the planet, which is pretty strong within the small distances at which they are revolving. I have seen small fixed stars, as near the planets as the satellites, and with no greater light, which, on removal of the planet, shone with a considerable lustre, such as I had by no means expected of them. A satellite of Jupiter, removed to the distance of the Georgian planet, would shine with less than the 180th part of its present light; and may we not conclude, that our new satellites would be of a very considerable brightness if they were brought so near as the orbit of Jupiter, and thus appeared

180 times brighter than at present? Nay, this is only when we take both the planets at their mean distance; for, in their oppositions, a satellite brought from the superior planet to the orbit of the inferior one, would reflect nearly 250 times the former light; from all which it is evident, that the Georgian satellites must be of a considerable magnitude.

If we draw together the results of the foregoing calculations into a small compass, they will stand as follows:

The first satellite revolves round the Georgian planet in 8 days 17 hours 1 minute and 19 seconds.

Its distance is $33''$.

And on the 19th of October, 1787, at 19 h. $11' 28''$, its position was $76^{\circ} 43'$ north-following the planet.

The second satellite revolves round its primary planet in 13 days 11 hours 5 minutes and 1,5 seconds.

Its greatest distance is $44'', 23$.

And on the 19th of October, 1787, its position at 17 h. $22' 40''$, was $76^{\circ} 43'$ north-following the planet.

Last year its least distance was $34'', 35$; but the orbit is so inclined, that this measure will change very considerably in a few years, and by that alteration we shall know which of the double quantities put down for the inclination and node of its orbit are to be used.

The orbit of the second satellite is inclined to the ecliptic $\left\{ \begin{array}{l} 99^{\circ} 43' 53,3'' \\ 81 \quad 6 \quad 4,4 \end{array} \right\}$.

Its ascending node is in $\left\{ \begin{array}{l} 18 \text{ degrees of Virgo} \\ 6 \text{ degrees of Sagittarius} \end{array} \right\}$.

When the planet passes the meridian, being in the node of this satellite, the northern part of its orbit will be turned towards the $\left\{ \begin{array}{l} \text{East} \\ \text{West} \end{array} \right\}$.

The situation of the orbit of the first satellite does not seem to differ materially from that of the second.

We shall have eclipses of these satellites about the year $\left\{ \begin{smallmatrix} 1799 \\ 1818 \end{smallmatrix} \right\}$, when they will appear to ascend through the shadow of the planet almost in a perpendicular direction to the ecliptic.

The satellites of the Georgian planet are probably not less than those of Jupiter.

The diameter of the new planet is 34217 miles.

The same diameter seen from the earth, at its mean distance, is $3'',90554$.

From the sun, at the mean distance of the earth, $1' 14'',5246$.

Compared to that of the earth as 4,31769 to 1.

This planet in bulk is 80,49256 times as large as the earth.

Its density as ,220401 to 1.

Its quantity of matter 17,740612 to 1.

And heavy bodies fall on its surface 18 feet 8 inches in one second of time.

W. HERSCHEL.

Slough, March 1, 1788.



Fig. 1.

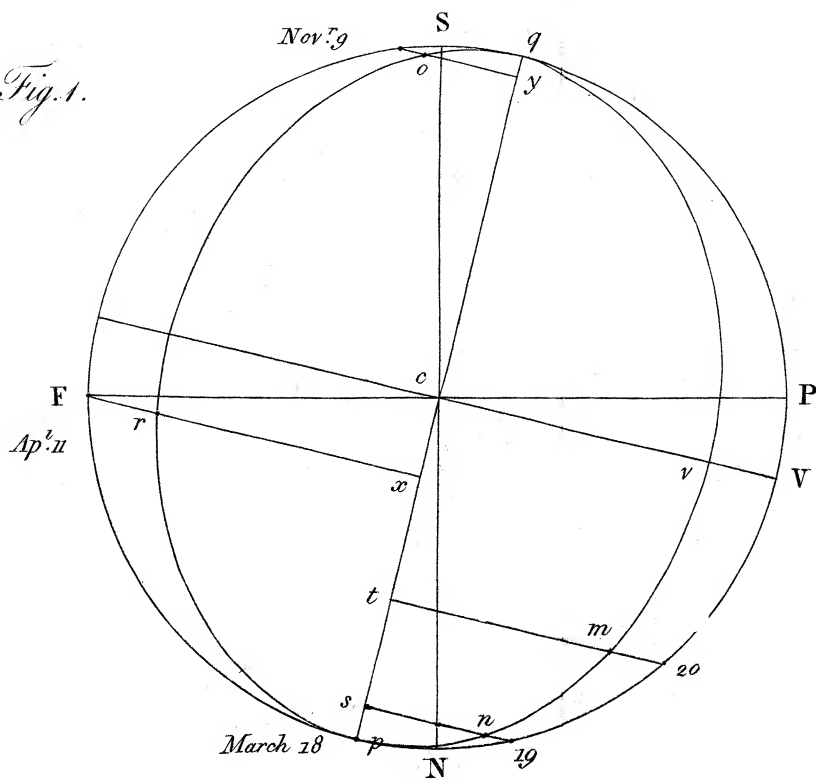


Fig. 2.

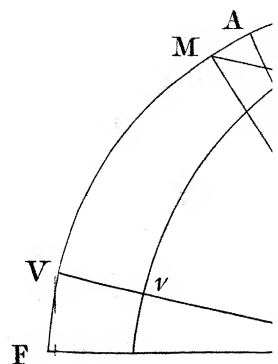


Fig. 3.

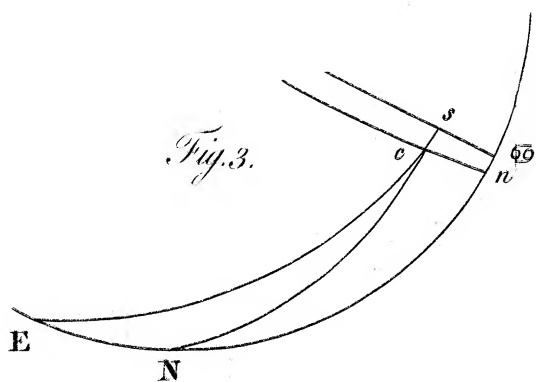
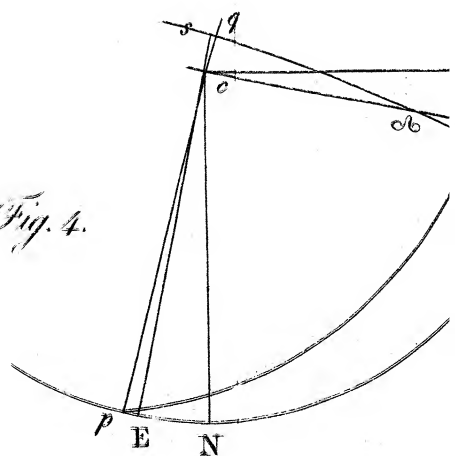


Fig. 4.



2.

